Rational Expressions,

Vertical Asymptotes, and Holes

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Rational Expression

- It is the quotient of two polynomials.
- A rational function is a function defined by a rational expression.

Examples:

$$f(x)=\frac{x-2}{x+5}$$

$$f(x)=\frac{3x^2+2x-5}{x^3+4x^2+5x-7}$$

Not Rational:

$$f(x)=\frac{4^x}{x+2}$$

$$f(x)=\frac{|x|}{x^2+5}$$

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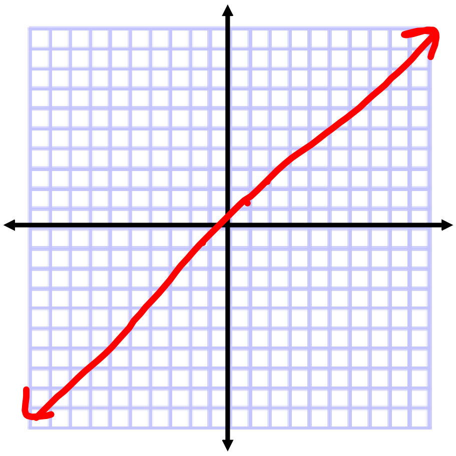
$f(x) = x$

$y = \frac{1}{x}$ inverse

graph it:



Write the domain

$(-\infty, \infty)$

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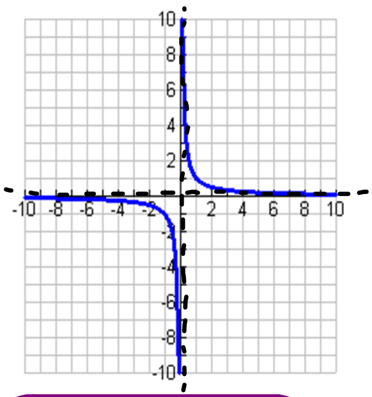
Parent Function

- The parent function is $\frac{1}{x}$
- The graph of the parent rational function looks like…
- The graph is not continuous and has asymptotes



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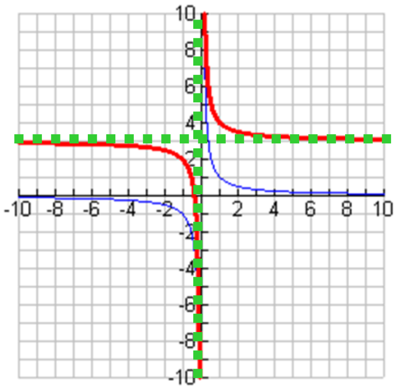
$$\frac{1}{x}+3$$



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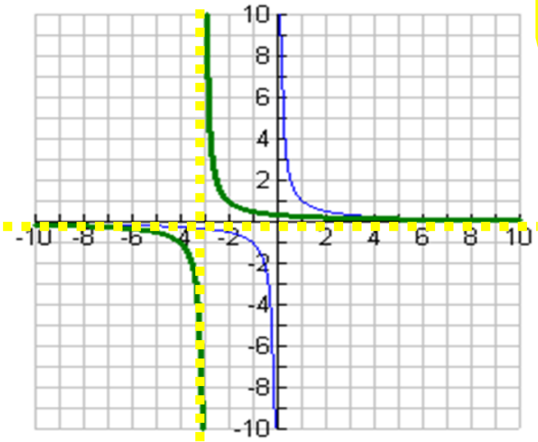
$$\frac{1}{(x+3)}$$



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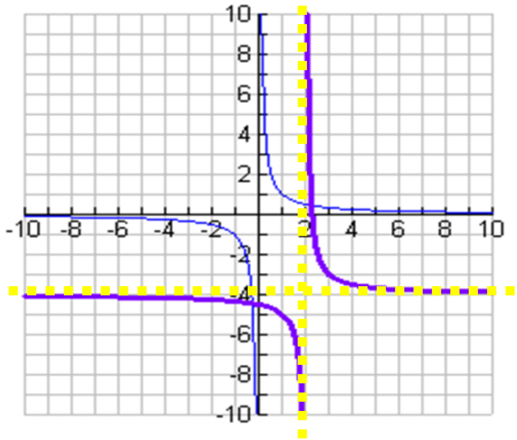
$$\frac{1}{(x-2)}-4$$



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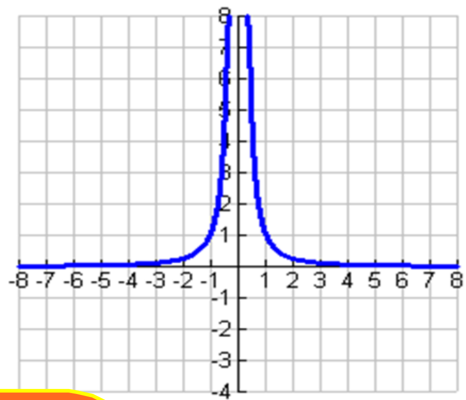
* All values are positive *

$$\frac{1}{x^2}$$



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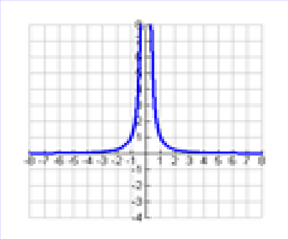
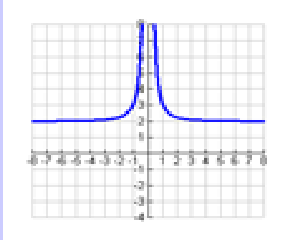
Transformations

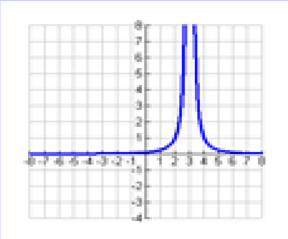
$\frac{1}{x^2}$



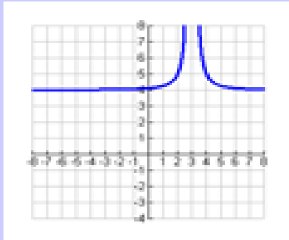
$\frac{1}{x^2} + 2$



$\frac{1}{(x-3)^2}$



$\frac{1}{(x-3)^2} + 4$



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What would cause $\frac{1}{x}$ to be undefined?

Conclusion:

If $x=0$, then $\frac{1}{x}$ is undefined.

Where would $\frac{1}{x+2}$ be undefined?

$x+2=0$ Set denominato = 0.

$x=-2$ Solve for x.

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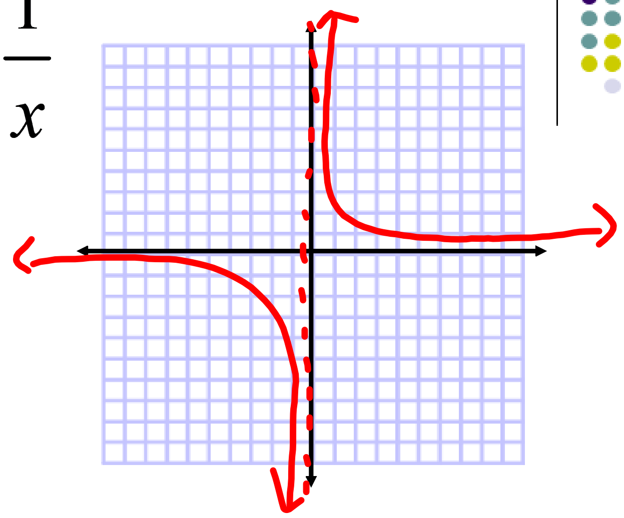
$$f(x)=\frac{1}{x}$$

x	y
-5	-1/5
-1	-1
-0.5	-2
-0.25	-4
-0.1	-10
-0.001	-1000
0	undefined
0.001	1000
0.1	10
1	1
5	1/5

Graph it:



Write the domain $(-\infty,0)\cup(0,\infty)$

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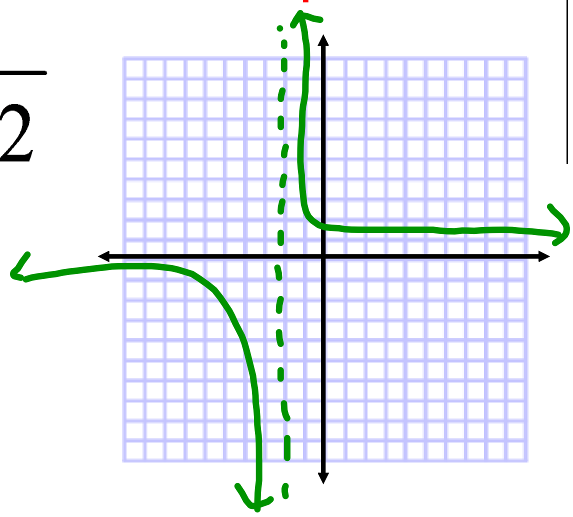
$$f(x)=\frac{1}{x+2}$$

x	y
-2	undefined
-1.75	4
-1.5	2
-1	1
0	1/2
1	1/3

Graph it:



Write the domain
$(-\infty,-2)\cup(-2,\infty)$

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Vertical Asymptote

- If (x – a) is a factor of the denominator of a rational function but not a factor of the numerator, then x = a is a vertical asymptote of the graph of the function.

$\frac{1}{x^2-4} = \frac{1}{(x-2)(x+2)}$

$x-2=0$
$x+2=0$

VA: $x=2$
$x=-2$

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$$f(x)=\frac{1}{(x+2)(x-3)}$$

$x+2=0$
$x-3=0$

where is this function undefined?

$x=-2$
$x=3$

Vertical Asymptotes? $x=-2$
$x=3$

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$$f(x) = \frac{\cancel{x-1}}{(x+4)\cancel{(x-1)}}$$

← x-1 is in both top & bottom.

x=1 is NOT a VA.

Where is this function undefined?

x=-4 x=1

Are they Vertical Asymptotes?

x=-4

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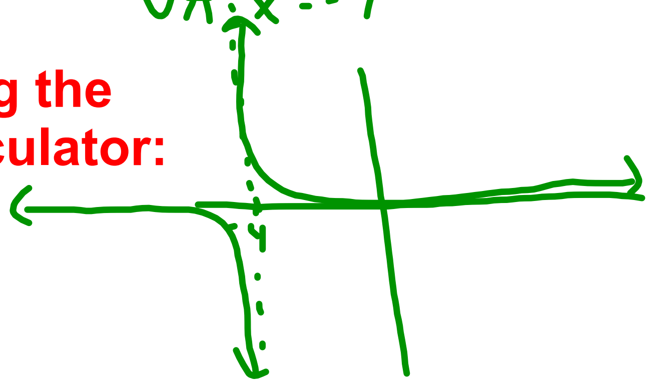
$$f(x) = \frac{x-3}{x^2+x-12} = \frac{x-3}{(x-3)(x+4)}$$

VA: x=-4



Graph it using the graphing calculator:

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Hole (in the graph)

- If (x – b) is a factor of both the numerator and denominator of a rational function, then there is a hole in the graph of the function where x = b.
- The exact point of the hole can be found by plugging b into the function after it has been simplified.

The numerator and denominator must be in factored form

$\frac{\cancel{x-1}}{(x+4)\cancel{(x-1)}}$

Hole: $x-1=0$

$x=1$

$\frac{1}{x+4}$ ← Simplified form

$\frac{1}{1+4}=\frac{1}{5}$

$\left(1,\frac{1}{5}\right)$

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$$f(x)=\frac{x-3}{x^2+x-12}$$

1. factor numerator and denominator

$\frac{\cancel{x-3}}{(x+4)\cancel{(x-3)}}$

$x-3=0$

$x=3$

Hole @ $\left(3,\frac{1}{7}\right)$

$\frac{1}{x+4}=\frac{1}{3+4}=\frac{1}{7}$

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Identify vertical asymptotes & holes.

Write the domain

$$f(x)=\frac{x+1}{x^2-2x-3}$$

VA: $x=3$

Hole @: $(-1, \frac{1}{-4})$

Domain: $(-\infty,-1)\cup(-1,3)\cup(3,\infty)$

$$\frac{\cancel{(x+1)}}{(x-3)\cancel{(x+1)}}$$

$x+1=0$

$x=-1$

$$\frac{1}{x-3}=\frac{1}{-1-3}=\frac{-1}{4}$$

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Identify vertical asymptotes & holes.

Write the domain

$$f(x)=\frac{x}{x^2+4}$$

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Identify vertical asymptotes & holes.

Write the domain

$$f(x)=\frac{x-5}{2x^2-x-3}$$

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Identify vertical asymptotes & holes.

$$f(x)=\frac{3-2x-x^2}{x^2+x-2}$$

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Homework

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