$$f(x) = 2x^5 - x^4 - 2x + 1$$

possible roots $\pm\frac{1}{1}, \pm\frac{1}{2}$

$\boxed{\pm 1, \pm\frac{1}{2}}$

1 is a root

$x=1$

1	2	-1	0	0	-2	1
	↓	2	1	1	1	-1
	2	1	1	1	-1	0

$(x-1)(2x^4+x^3+x^2+x-1)$ factors

List all possible rational roots.

$$f(x) = 12x^5 + 6x^4 - 7x^3 + 2x^2 - 3x + 18$$

Find all possible rational roots and then find ones that work.

$$f(x) = 2x^3 - 11x^2 - 4x + 9$$

$\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}, \pm 9, \pm \frac{9}{2}$

① List the possible rational roots

$2(-1)^3 - 11(-1)^2 - 4(-1) + 9 = 0$

② Plug the possible roots into the eq. until you find one that has an answer of zero

$$\begin{array}{r|rrrr} -1 & 2 & -11 & -4 & 9 \\ & \downarrow & -2 & 13 & -9 \\ \hline & 2 & -13 & 9 & 0 \\ & a & b & c & \end{array}$$

$2x^2 - 13x + 9$

③ Use the root that you found to do synthetic div.

$$\frac{13 \pm \sqrt{169 - 4(2)(9)}}{2(2)}$$

④ Solve the Quadratic
- factor
- Quadratic formula

$$\boxed{x = \frac{13 \pm \sqrt{97}}{4}, -1}$$

⑤ List ALL of the roots (don't forget the one you found 1st)

$$f(x) = 2x^5 - x^4 - 2x + 1$$

possible roots $\pm\frac{1}{1}, \pm\frac{1}{2}$

$\boxed{\pm 1, \pm\frac{1}{2}}$

1 is a root

$x = 1$

$$\begin{array}{r|rrrrrr} 1 & 2 & -1 & 0 & 0 & -2 & 1 \\ & \downarrow & 2 & 1 & 1 & 1 & -1 \\ \hline & 2 & 1 & 1 & 1 & -1 & 0 \end{array}$$

$(x-1)(2x^4+x^3+x^2+x-1)$ factors

$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & 1 & 1 & 1 & -1 \\ & \downarrow & 1 & 1 & 1 & 1 \\ \hline & 2 & 2 & 2 & 2 & 0 \\ & \div 2 & \div 2 & \div 2 & \div 2 & \end{array}$$

x^3+x^2+x+1

$$\begin{array}{r|rrrr} -1 & 1 & 1 & 1 & 1 \\ & \downarrow & -1 & 0 & -1 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$x^2 + 1 = 0$ Square root method

$\sqrt{x^2} = \sqrt{-1}$

$x = \pm i$

$\boxed{x = \pm i, -1, \frac{1}{2}, 1}$

Rational Root Theorem

Solve. $f(x) = x^3 - 4x^2 + 6x - 4$

List the roots and then list all of the factors

Conjugate Root Theorem

If P(x) is a polynomial with REAL coefficients, then irrational roots of P(x) occur in conjugate pairs.

So...

If $\sqrt{3}$ is a root, then $-\sqrt{3}$ must be a root

If $2-\sqrt{5}$ is a root, then $2+\sqrt{5}$ must be a root

$-3+\sqrt{6}$ $-3-\sqrt{6}$

Complex Conjugate thm

If there is an imaginary (complex) answer, then its conjugate is also an answer.

$2i$, $-2i$

$3+5i$, $3-5i$

Find the lowest degree polynomial with the following roots

3 and $-2i$.

$x = 3$ $\quad x = -2i$ $\quad x = 2i$

$+2i \quad +2i \qquad -2i \quad -2i$

$(x-3)\ \underbrace{(x+2i)(x-2i)}$

① Write the roots as $x =$

(Remember the conjugate thms)

② Bring the roots back over & write the factors.

③ Multiply

Begin w/ complex or $\sqrt{\ }$

$(x-3)(x^2 \cancel{-2xi} \cancel{+2xi} - 4i^2)$

$-4 \cdot -1$

$(x-3)(x^2+4)$

$x^3 + 4x - 3x^2 - 12$

$\boxed{x^3 - 3x^2 + 4x - 12}$

$$3, -5, \sqrt{6}$$

$x = 3 \quad x = -5 \quad x = \sqrt{6} \quad x = -\sqrt{6}$

$\underbrace{(x-3)(x+5)}\underbrace{(x-\sqrt{6})(x+\sqrt{6})}$

$(x^2+2x-15)(x^2-6)$

$x^4 - 6x^2 + 2x^3 - 12x - 15x^2 + 90$

$x^4 + 2x^3 - 21x^2 - 12x + 90$